

mqual: A program for Bock's Multinomial Response Model

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Why Use Bock's Model?

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program for
Bock's
Multinomial
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Model

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- Bock (1970, 1975) was early innovator in developing ML methods for categorical data
- Today we have much fancier methods (e.g. glm in R!)
- Why I like Bock's model: Data structure
- Indep vars \implies Dep vars
- Most social science analyses have this structure

Example Data Set

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Sample		Response		
Gender	Race	Democrat	Republican	Independent
male	white	132	176	127
male	black	42	6	12
female	white	172	129	130
female	black	56	4	15

- Sample design is 2 (sex) \times 2 (race)
- Response design is 3 (party affiliation)
- Of interest: Relation of sample variables to response
- Note: small number of blacks may lead to low power, collinearity problems

Example Data Set: Row Proportions

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Sample		Response		
Gender	Race	Democrat	Republican	Independent
male	white	0.303	0.405	0.292
male	black	0.700	0.100	0.200
female	white	0.399	0.299	0.302
female	black	0.747	0.053	0.200

- Natural sample questions (and contrasts):
 - Is sex related to party affiliation?
 - Is race related to party affiliation?
 - Is there a sex x race interaction?
- Natural response questions (and contrasts):
 - Is the effect on party affiliation vs independent?
 - For party members, is there an effect on Dem v Rep?

Example Data Set: Row Log Proportions = $\ln(p)$

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Sample		Response		
Gender	Race	Democrat	Republican	Independent
male	white	-1.193	-0.905	-1.23
male	black	-0.357	-2.303	-1.61
female	white	-0.919	-1.206	-1.20
female	black	-0.292	-2.931	-1.61

Why Logarithms of Proportions?

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- Contrasts among $\ln(\pi_i)$ generalize the usual logit
- $\text{logit}(\pi) = \ln\left(\frac{\pi}{1-\pi}\right)$
- $= \ln(\pi) - \ln(1 - \pi)$

Model for Multinomial Logits

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- $Z = KBT$ (Analogous to usual MANOVA model)
- Z : expected values of the multinomial logits
- K : model matrix for the sample variables
- B : parameters
- T : model matrix for response variables
- Element i in each row of Z is related to the corresponding probability:

$$\pi_i = \exp(z_i) / \sum_j \exp(z_j)$$

Example of Sample and Response Matrices

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- K matrix for a saturated model:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

- T matrix for a saturated model:

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$$

Parameter Estimates from Saturated Model

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Parameter Estimates

	DR v I	D v R
intcpt	0.049679632	0.57312093
male	0.027364329	-0.15856392
white	0.003416179	-0.57312093
m.w	-0.019641516	0.01472288

Parameters/SE

	DR v I	D v R
intcpt	1.11308417	6.5825656
male	0.61310442	-1.8211817
white	0.07654031	-6.5825656
m.w	-0.44007291	0.1690993

Parameter Estimates from Main Effects Model

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Degrees of Freedom = 2
Likelihood Ratio Fit Statistic = 0.1982117
Probability(LR) = 0.9056468

Parameter Estimates

	DR v I	D v R
intcpt	0.049685973	0.5699511
male	0.011032223	-0.1431899
white	0.003463126	-0.5695336

Parameters/SE

	DR v I	D v R
intcpt	1.12450050	6.646813
male	0.46869301	-3.636090
white	0.07846031	-6.645789

Preparing Data Matrix

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```
# number of people in each party, by group
```

```
dem <- c(132, 42, 172, 56)
```

```
rep <- c(176, 6, 129, 4)
```

```
ind <- c(127, 12, 130, 15)
```

```
party.f <- cbind(dem,rep,ind)
```

Prepare Contrast Matrices

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```
const <- c(1,1,1,1)      # intercept
ge <- c(1,1,-1,-1)      # gender
ra <- c(1,-1,1,-1)      # race
ge.ra <- ge * ra        # gender x race

party.k <- cbind(const,ge,ra,ge.ra)

dr.i <- c(1,1,-2) # Party member vs independent
d.r <- c(1,-1,0) # Dem vs Rep for party members

party.t <- rbind(dr.i,d.r)
```

Provide Names (optional) and Run

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```
rownames(party.f) <- c("male white  ",  
                        "male black  ",  
                        "female white",  
                        "female black")
```

```
colnames(party.f) <- c("Dem", "Rep", "ind")
```

```
rownames(party.t) <- c("DR v I", "D v R")
```

```
colnames(party.k) <-  
  c("intcpt", "male", "white", "m.w")
```

```
mqual(party.f, party.k, party.t)
```

Example: Nonstandard Loglinear Model

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Mother's preference	Father's preference	Child's Preference	
		Johnson	Goldwater
Johnson	Johnson	256	18
	Goldwater	14	7
Goldwater	Johnson	20	5
	Goldwater	45	93

- Is mother's preference predictive of child's?
- Is father's preference predictive of child's?
- Do the parents have equal influence on child?

Main effects model

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- Sample model matrix K:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

- Response model matrix T:

$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

- Model fits well: $LR = .041, df = 1, p = .839$

Are effects of parents equal?

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- Add columns 2 and 3 of K together (see Rindskopf, 1984):

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

- Nonstandard loglinear model (Rindskopf, 1990)
- Equivalent to: Number of parents favoring Johnson
- Model fit is excellent: $LR = 1.095, df = 2, p = 0.578$
- Parents have equal effect on children's preference

Summary: Why Use Bock's Model?

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- Data structure: IVs \implies DVs
- Corresponds to many social science situations
- Parallel to MANOVA model for continuous variables
- Allows/encourages coding of special (nonstandard) models

References

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- Bock, R. D. (1970). Estimating multi-nominal response relations. In R. C. Bose, et al. (Eds.), *Essays in probability and statistics*. Chapel Hill: University of North Carolina Press.
- Bock, R. D. (1975). *Multivariate statistical methods in behavioral research*. New York: McGraw-Hill.
- Bock, R. D., & Yates, G. (1973). *MULTIQUAL: Log-linear analysis of nominal or ordinal qualitative data by the method of maximum likelihood*. Chicago: National Educational Resources, Inc.
- Rindskopf, D. (1984). Linear equality restrictions in regression and loglinear models. *Psychological Bulletin*, *96*, 597-603.
- Rindskopf, D. (1990). Nonstandard log-linear models. *Psychological Bulletin*, *108*, 150-162.