Estimating a Spatial Filtering Gravity Model for Bilateral Trade: Functional Specifications and Estimation Challenges

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Based on:

Linders G.-J. and R. Patuelli, The Space of Gravity: Spatial Filtering Estimation of a Gravity Model for Bilateral Trade (in progress)

Content

- Introduction: gravity model and trade
- Estimation of a cross-section gravity model
- The spatial filtering solution
- Estimation in `R`
  - Overdispersion adjustment choices and possible estimation approaches
  - Estimation problems (and remedies?)
- Conclusions

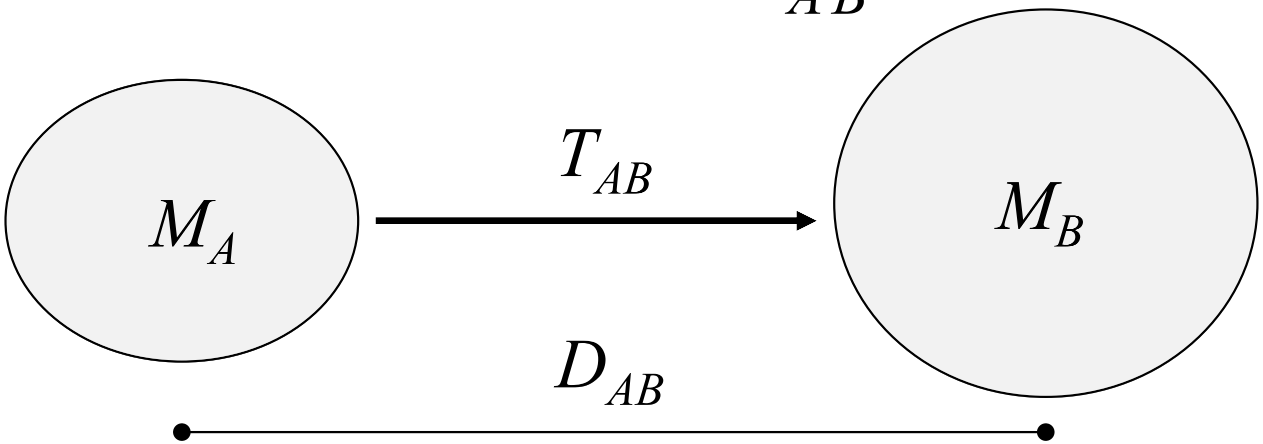
Gravity/Spatial-Interaction Model

- Different purposes:
  - Parameter estimation to describe interaction patterns
  - Predicting flows in response to changing variables or parameters (‘comparative statics’)
  - Estimating current flows and predicting future flows
- Three generic types of models
  - Unconstrained
  - Singly-constrained (one constraint)
  - Doubly-constrained (both constraints hold)
- Choice of model depends on purpose

Unconstrained Gravity Model

$$T_{ij} = O_i \cdot D_j \cdot f(d_{ij})$$

$$T_{AB} = G \frac{M_A M_B}{D_{AB}^2}$$



- Criticized for leading to biased parameter estimates because of omitted variable bias (Anderson and van Wincoop 2003)

Doubly-Constrained Gravity Model

- Satisfies both restrictions on incoming and outgoing flows
- Combines both outward and inward accessibility

$$T_{ij} = A_i \cdot O_i \cdot B_j \cdot D_j \cdot f(d_{ij})$$

  – where: $A_i = \dfrac{1}{\sum_k B_k \cdot D_k \cdot f(d_{ik})}$ $\quad B_j = \dfrac{1}{\sum_l A_l \cdot O_l \cdot f(d_{lj})}$

- But requires iterative estimation of balancing factors A and B
  – Can it be done easily in `R`? (open question)

Alternative uses

- The gravity model is used also for the study of (inter alia):
  - Commuting
  - Migration
  - Patent citations
  - Many more… (basically any flow data model based on origin/destination characteristics and bilateral/deterrence variables)

Gravity model of trade

- Analysis of patterns of bilateral trade
- Focus on, for example:
  - Border effect and intangible trade barriers
  - Trade creation and diversion by free trade agreements
  - Estimation methods
- Empirical gravity model of trade
  - bilateral independents (language, distance, …)
  - country-specific regressors (market size, island, landlocked)
- Problems:
  - Consistent estimation
  - Prediction and comparative statics (e.g. trade liberalization)

Estimation issues: balancing factors

- Problem
  - The theory-derived gravity model for bilateral trade should include multilateral resistance terms (balancing factors, ‘remoteness’ variables or ‘price indices’) to correct for the effect of inward and outward accessibility (i.e., multilateral trade costs?)
- Estimating cross-sectional model:
  - Non-linear equations (e.g. Anderson and van Wincoop 2003)
  - Dummies for importers/exporters → but no parameters can be estimated for origin- or destination-specific variables
    - Baier and Bergstrand (2009): estimating with ‘good old’ OLS (loglinearized multilateral resistance terms using a first-order Taylor series approximation)

Estimation issues: distribution

- Problems
  - Trade flows are non-negative (all flows > 0), and can be treated as count variables. The log-linearization can bias some estimates in presence of heterogeneity (Santos Silva and Tenreyro 2006)
  - Trade data are usually strongly overdispersed (`dispersiontest` from `AER` package)
  - The matrix of trade flows contains a high share of zeros
- Solutions
  - Count data models are more suitable for trade data (Santos Silva and Tenreyro 2006)
    - Consequent problem: most advanced estimation methods proposed are not fit for Poisson regressions (e.g. Beherens et al. 2007)
  - Overdispersion
    - Overdispersed Poisson (`glm.poisson.disp` from `dispmod` package)
    - Quasi-poisson (within `glm`)
    - Negative binomial (`glm.nb` from `MASS` package)
  - Excess zeros ignored at this stage (`zeroinfl` from `pscl` package)
    - But how to model the zero? Replicating the same covariaties seems just lazy

Estimation issues: spatial correlation

- Problem
  - Beherens et al. (2007) argue that the fixed-effects specification does not fully capture MR dependencies in the error structure introduced by the general equilibrium nature of trade patterns modelling → which would violate independence assumption
  - This problem largely neglected in flow data models
  - The unconstrained gravity model overlooks spatial autocorrelation (SAC) underlying the distribution of origin- and destination-specific variables (as well as network autocorrelation)
- Proposed solution
  - In Linders et al. (2009), we propose to use spatial filtering to correct for spatial autocorrelation in origin- and destination-specific data
  - We consider the count nature of the data, as well as its overdispersion
  - The obtained spatial filters (orig. and dest.) behave as two sets of indicator variables (which themselves correspond to the doubly-constrained balancing factors)
  - Possibility of estimating parameters of origin- and destination-specific variables
  - Considerable savings in terms of degrees of freedom
  - Can help accounting for heterogeneity

Spatial filtering

- Does not directly model SAC (a la spatial Durbin-type models), but rather corrects for it
  - For flow data, compare to spatial econometric interaction models (Beherens et al., Fischer and Griffith, LeSage and Llano, LeSage and Pace, LeSage and Polasek)
- Assumes that SAC among flow residuals is induced by omitted (spatially autocorrelated) origin and destination variables.
- Based on eigenvectors decomposition techniques...
  - Similarities with principal components analysis
- ... and on the computational formula of Moran's I

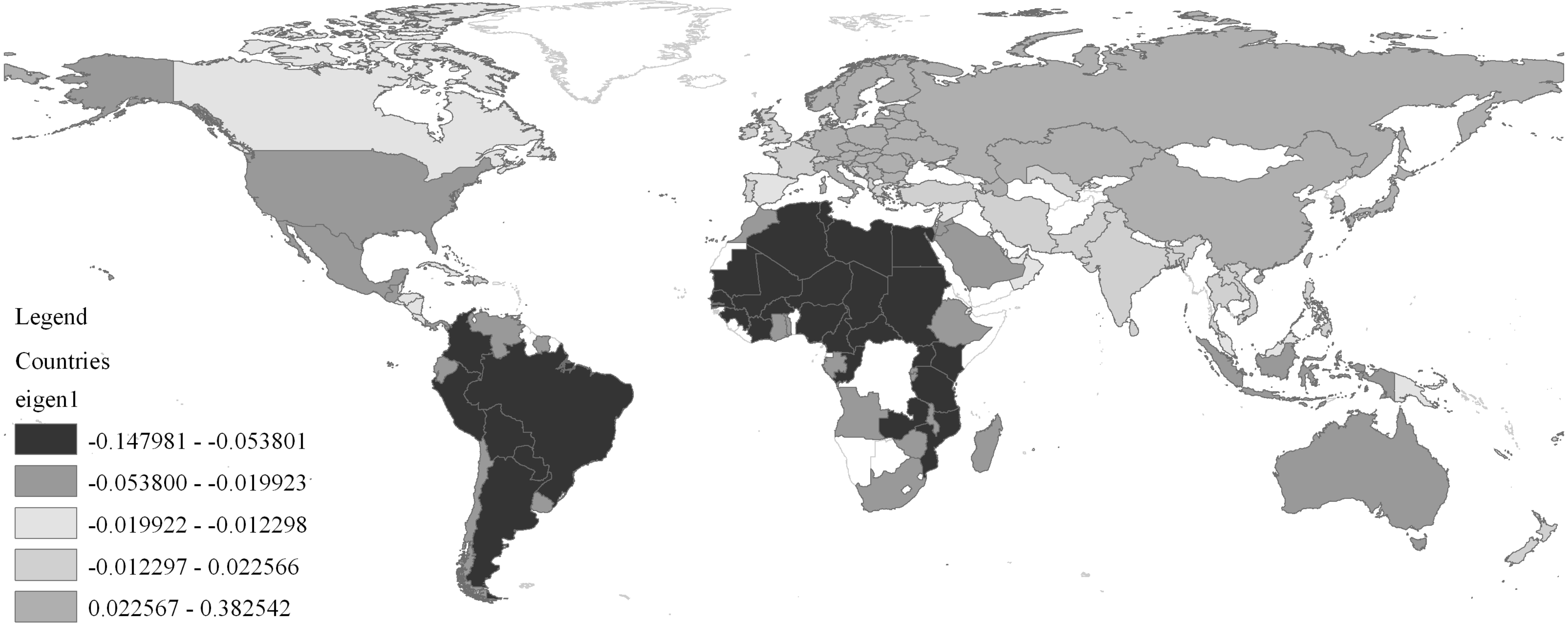
$$I = \frac{N \sum_i \sum_j w_{i,j} (x_i - \overline{x})(x_j - \overline{x})}{(\sum_i \sum_j w_{i,j}) \sum_i (x_i - \overline{x})^2},$$

- Non-parametric techniques that removes inherent SAC by introducing surrogate variates (eigenvectors) for the omitted variables
- Provided as function `SpatialFiltering` in the `spdep` package, but the flow data case requires modifications

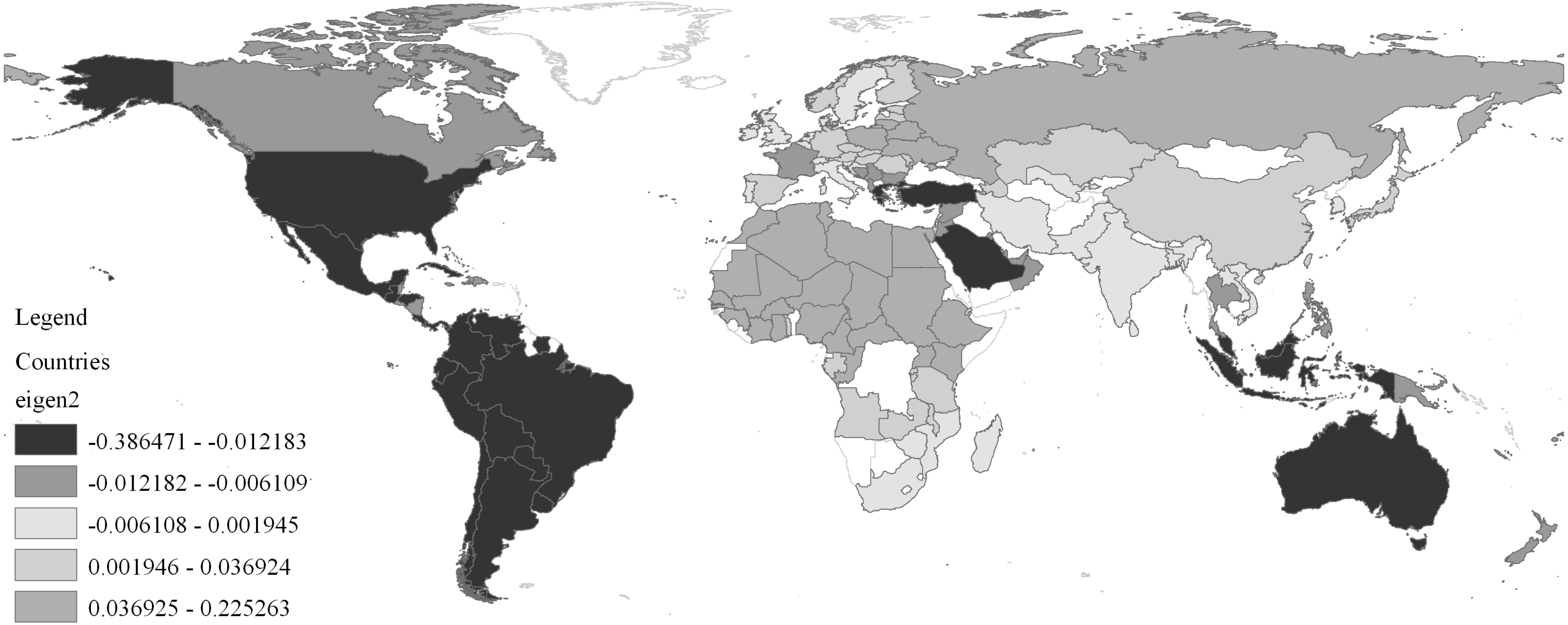
Spatial filtering (2)

- Step 1: define an *n* x *n* spatial weights matrix **W**:
  - We define proximity by means of (rook) contiguity
- Step 2: transform **W**:
  - $(\mathbf{I} - \mathbf{11}^T/N)\mathbf{W}(\mathbf{I} - \mathbf{11}^T/N)$ which appears in the numerator of MI
- Step 3: extract eigenvectors (`eigen`)
  - Because of the transformation, the eigenvectors map spatial patterns implied by the contiguity relations between countries/regions (no relation to the data still)
  - First selected eigenvector has highest SAC
  - Following eigenvectors maximize SAC while being orthogonal to previous ones
- Step 4: Select SAC-relevant eigenvectors (*candidate eigenvecs*)
  - MI_j / $\max_j (MI_j) > 0.25$ threshold (roughly corresponds to 95% sign. in SAR model). Function `moran.test` from `spdep` package gives MI score
  - Choice of whether or not to include negative MI eigenvecs
- At this point, we have a set $\mathbf{E}_k$ of k ($< n$) candidate eigenvectors. How do we relate them to the data?

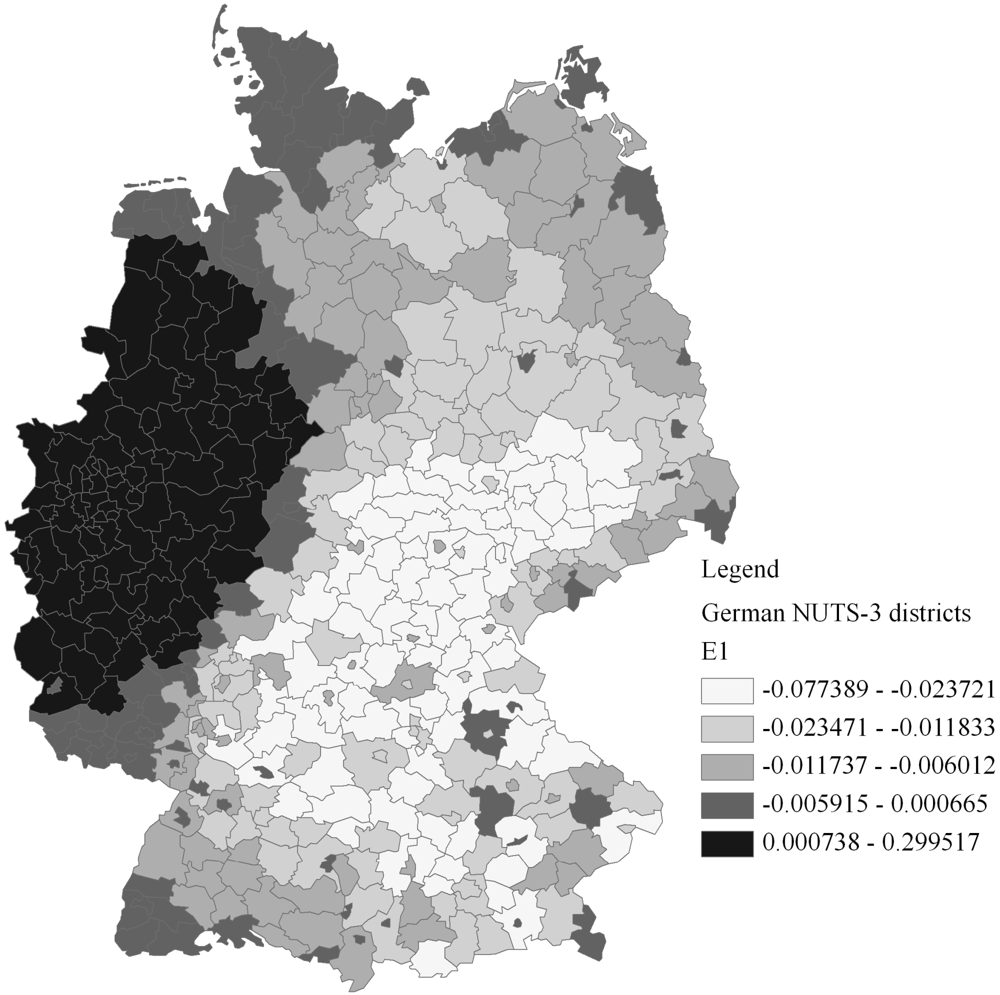
First extracted eigenvector

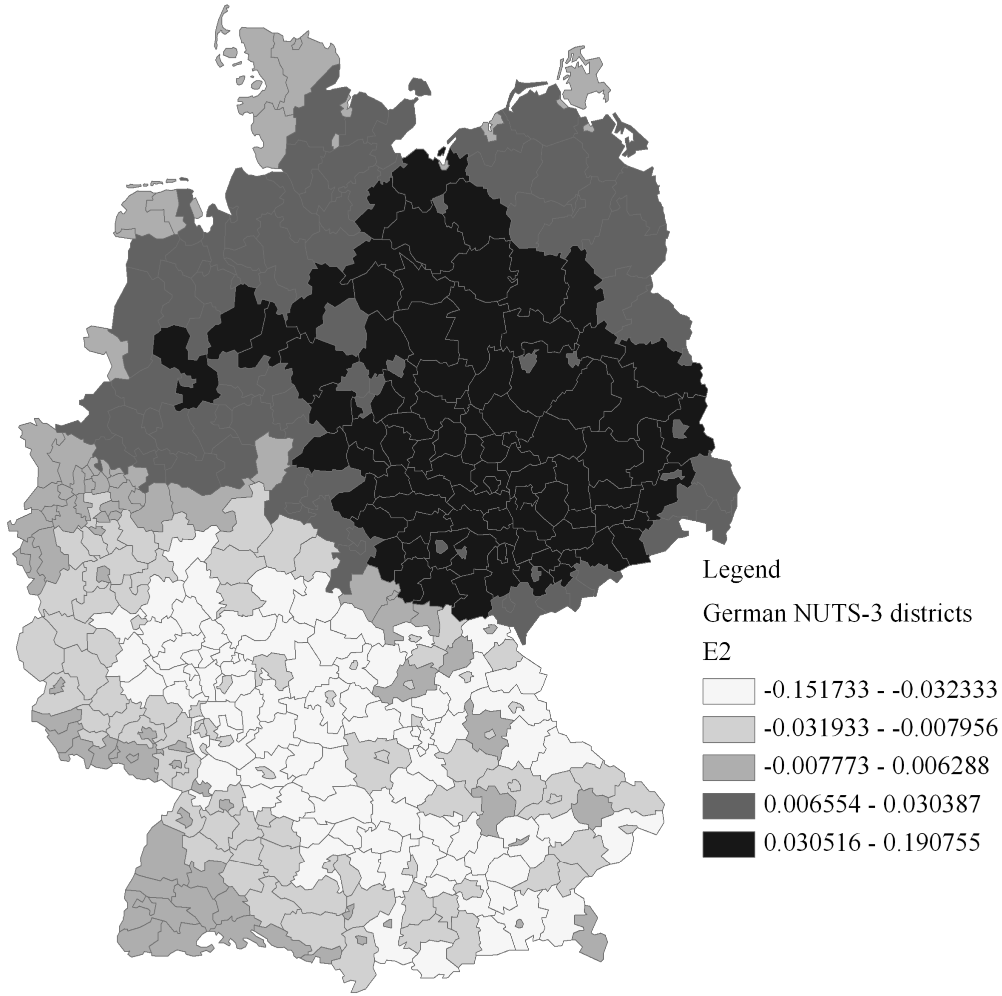


Second extracted eigenvector



Example: first- and second-extracted eigenvectors for German rook contiguity





Spatial filtering (3)

- Estimate a generalized linear model, e.g., a Poisson regression
- Generate orig.- and dest.-specific eigenvectors
  - Kronecker products $\mathbf{E}_k \otimes \mathbf{1},\ \mathbf{1} \otimes \mathbf{E}_k$
    - `kronecker(matrix(1, n, 1), eigenvecs)`
    - `kronecker(eigenvecs, matrix(1, n, 1))`
- Select eigenvectors by means of stepwise regression (`step`) (all candidate eigenvectors are orthogonal and independent)
- The two linear combinations of the sets of selected eigenvectors (for orig. and dest.) are called our ‘spatial filters’
  - They incorporate spatial structuring into the intercept term
  - The T_{ij}-specific spatial filter contribution is $sf_{Ai} + sf_{Bj}$

Empirical application

- Bilateral trade data for 137 countries, one cross-section (year 2000)
  - Dependent variable is flows value (in million US$)
  - Standard explanatory variables for trade models, both importer/exporter-specific and bilateral
- **W** matrix based on rook contiguity (common border)
  - Main sea routes (as shown on Google Maps) used for islands
  - More definitions of proximity to be tested, such as *k*-nearest neighbours and distance decay
- We estimate a quasi-poisson model, which takes into account overdispersion
- Two benchmark models (not shown here)
- Selected eigenvectors: 21 for origins, 25 for destinations
- Compare orig./dest. spatial filters and fixed effects from traditional estimation

	(1)	(2)	(3)	(4)
	Fixed effects (GDP offset)	Spatial filter	BB-estimation (GDP offset)	BB-estimation
Distance	–0.63***	–0.61***	–0.54***	–0.52***
Common border	0.59***	0.58***	0.90***	0.76***
Common language	0.10	0.16***	0.32***	0.23**
Common history	0.15*	0.18***	–0.03	0.08
Free trade	0.43***	0.41***	0.41***	0.55***
GDP exporter		0.81***		0.78***
GDP importer		0.82***		0.82***
GDP per cap. exporter		0.01	–0.22***	–0.03
GDP per cap. importer		0.00	–0.10***	0.05*
Island exporter		–0.20***	–0.12	–0.22***
Island importer		0.12***	0.03	–0.06
Area exporter		–0.05***	–0.18***	–0.07***
Area importer		–0.14***	–0.16***	–0.08***
Landlocked exporter		–0.23***	0.01	–0.15
Landlocked importer		–0.34***	0.09	–0.01
Constant	–35.23***	–29.31***	–32.40***	–27.06***
Adj. pseudo R^2	0.792	0.930	0.604	0.916
AIC	1.97e+09	n.a.	3.80e+09	3.40e+09
Observations	18632	18632	18632	18632

Estimation in `R`: Issues

- Approach used: quasi-poisson estimation
  - Problem: `step` cannot be used for selecting eigenvectors because of missing likelihood
  - Solutions:
    - 'Manual' stepwise, based on χ^2 values. But is it appropriate? (and it can probably be automated)
    - Stepwise based on different criteria (for example, minimization of SAC, see Chun and Griffith 2008)
- Why not overdispersed Poisson or negative binomial?
  - They would not converge (getting trapped between two local minima?)
  - Currently testing if different `init.theta` values may help avoiding this in `glm.nb`
- Why not zero-inflated negative binomial (`zeroinfl`) or hurdle model (`hurdle`)?
  - Currently testing the 'lazy' solution (same regressors for both zeros and count). But scarce attention to theoretical considerations will be criticized… (in particular for the hurdle model, which is a two-part model)
  - Can it be "stepwised" through `step`? Apparently yes…

Estimation in `R`: Issues (2)

- Why not tobit model?
  - Gaussian underlying distribution does not seem to make sense for count (and overdispersed) data
- More generally, problem with `step`
  - It tends to overselect (see also D. Hendry's critiques), forcing to manual backward elimination so to have all selected eigenvecs at least 95% sign.
  - Can it be automated? (without discarding 'real' variables)

Conclusions and Current `R` Issues

- Estimation of a gravity model of trade taking into account:
  - Count nature of the data
  - Overdispersion
  - Spatial dependence in origins and destinations
  - Excess zeros? (currently being tested)
- The constructed spatial filters represent spatial dependence in origins and destinations, as the balancing factors or the dummy estimates (but in the dummy estimates also non-significant dummies are used)
- `R`-related issues
  - Quasi-poisson estimation does not allow for stepwise regression
  - Convergence problems (dispersion estimatation) with overdispersed Poisson and negative binomial
  - Zero-inflated negative binomial and hurdle models feasible, but (a) can they be ‘stepped’? (b) theoretical model concerns
  - Tobit not fit for count data
  - `step` function tends to overselect, forcing to manual ‘cleaning’